Precalculus, Quarter 1, Unit 1.1

Parent Functions and Transformations

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Compare and contrast graphs of parent functions and transformations.
- Write the equations for the transformation of the graph.
- Recognize even or odd functions and their graphs.
- Translate parent functions horizontally and vertically.
- Stretch and shrink parent functions.
- Reflect parent functions.
- Use technology to graph parent functions and transformations.

Mathematical practices to be integrated

Model with mathematics.

- Graph, transform, and describe parent functions on whiteboards or paper.
- Using given equations, identify the parent function and graph the transformations.

Use appropriate tools strategically.

- Use graphing calculators to graph parent functions and transformations.

Look for and make use of structure.

- Describe vertical translations, horizontal translations, reflections, stretches, and shrinks on parent functions.

Essential questions

- What does a restricted domain mean for a function?
- How do you prove that one function is an inverse of another?
- What are the similarities and differences between the transformations of various functions?
- How are the multiple transformations of a function related?
- What does the characteristic of being even or odd tell you about a function?

Written Curriculum

Common Core State Standards for Mathematical Content

Building Functions	**F-BF**

Build new functions from existing functions [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Learning about transformations of functions began in kindergarten. Shapes were identified and described. In grades 1–3, shapes were drawn with defining attributes. Graphing of points on the coordinate plane was mastered in grade 5. Geometrical figures were drawn, and constructed and the relationship between the figures was described during the seventh grade.

During eighth grade, the properties of rotations, reflections, and translations were verified. The concepts of rotation, reflection, and translations will be extended in geometry with precise definitions. Definitions of trigonometric ratios are mastered during geometry.

Relationships between sine and cosine of complementary angles are explained and used. In algebra I, functions were built from information or provided data. The functions built will include linear, exponential, quadratic, and absolute value. New functions were constructed from existing function. Some included functions are simple radicals, rational, and exponential functions. Transformations across the functions types will be emphasized. Transformations will have been demonstrated manually and with the use of the graphing calculator.

Current Learning

Students will graph the parent functions (linear, quadratic, simple radical, rational, and exponential) then do the translations. Using technology, students will experiment with the graphs of sine, cosine, tangent, secant, cosecant, and cotangent. In this course, students extend transformations and the building of new functions to include more complex polynomial and absolute value functions. They write equations for more complex radical, rational, and exponential functions. They extend their fluency with inverses by working interchangeably with equations, tables, and graphs.

Future Learning

These concepts will be used in calculus, trigonometry, and physics. Management occupations that will be using graphs of trigonometric functions include engineering and natural sciences managers, farmers, ranchers, and agricultural managers, funeral directors, industrial production managers, medical service managers, property, real estate, and community association managers, purchasing managers, buyers, and purchasing agents.

Insurance underwriter and other business and financial operations occupations will find trigonometric functions beneficial. Graphs of trigonometric functions will also prove useful to forest, conservation, and logging workers. Interested in installation careers? Electronic installation and repair will be making use of graphs of trigonometric functions. Electricians will need these functions for training. Engineers will make use of graphs of trig functions

Additional Findings

Transformations of functions can cause some students to struggle if they do not realize that the behavior of all functions is the same. Students can use technology, such as graphing calculators to represent and study the behavior of functions such as polynomial, exponential, rational, and periodic functions. “In high school, students should have opportunities to build on these earlier experiences, both deepening their understanding of relations and functions and expanding their repertoire of familiar functions.” (*Principles and Standards for School Mathematics*, p. 297)

Precalculus, Quarter 1, Unit 1.2

Rational Expressions, Functions, and Graphs

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Analyze multiple representations of rational functions.
- Find characteristics of simple rational functions without technology and with technology.
- Perform operations on rational expressions, and use those operations to solve rational equations.

Mathematical practices to be integrated

Model with mathematics.

- Model a rational function with an age problem.

Use appropriate tools strategically.

- Use graphing technology, white boards, and a straight edge to construct rational functions.

Attend to precision.

- Calculate the vertical and horizontal asymptotes.

Essential questions

- What are the key characteristics of a rational function?
- How are fractions and rational functions similar and different?
- What are the different processes involved in adding or subtracting rational expressions with like versus unlike denominators?
- When can you cancel diagonally versus vertically in a rational expression/equation?
- How do rational functions apply in the real world?
- What are the methods for finding vertical, horizontal, and slant asymptotes?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions	**F-IF**

Analyze functions using different representations [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Arithmetic with Polynomials and Rational Expressions	**A-APR**

Rewrite rational expressions [*Linear and quadratic denominators*]

A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

Rational functions are developed as early as the grade 3. Students explored fractions and then perform operations with fractions in grades 4 and 5. In sixth grade, students learned the concept of a ratio as well as use proportional reasoning to solve problems. In grade 7, students continue using operations with fractions. In eighth grade, students discovered rational and irrational numbers. In algebra 1, students worked with the key features of linear and exponential functions (intercepts, zeros, maxima, minima, end behavior, symmetries, increasing and decreasing intervals, positive and negative intervals, and periodicity) using a modeling approach. In geometry, students used trigonometric ratios to solve right triangles.

In algebra 2, students focused on applications and how key features relate to characteristics of situations, selecting the appropriate type of function model. The types of functions involved were graphing square root, cube root, piecewise including step functions, absolute value, exponential, and logarithmic functions. In algebra 2, students, also, rewrite rational expressions and also combine rational functions.

Current Learning

The concept of a rational function is reinforced and students graph rational functions expressed symbolically, by hand and using technology. Students also identify zeros and horizontal and vertical asymptotes, as well as show end behavior.

Future Learning

In future courses rational functions will be used in calculus AB and college algebra. In calculus AB students will study limits, which are the foundation for derivatives and integrals. There are many applications in finance, demographics, and so on where it is useful to know what happens in the far future.

Additional Findings

Students often struggle with "seeing" how a function can model a real-world problem. Using technology, teachers can help students model and understand mathematical concepts. "With utilities for symbol manipulation, graphing, and curve fitting and with programmable software and spreadsheets to represent iterative processes, students can model and analyze a wide range of phenomena. These mathematical tools can help students develop a deeper understanding of real-world phenomena. At the same time, working in real contexts may help students make sense of the underlying mathematical concepts and may foster an appreciation of those concepts . . . In helping high school students learn about the characteristics of particular classes of functions, teachers may find it helpful to compare and contrast situations that are modeled by functions from various classes." (*Principles and Standards for School Mathematics*, p. 297)

Precalculus, Quarter 1, Unit 1.3

Logarithmic and Exponential Equations, Functions, and Graphs

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Analyze functions using different representations.
- Graph exponential and logarithmic functions by hand.
- Understand exponential and logarithmic functions are inverses.
- Solve problems using exponential and logarithmic functions.
- Graph exponential and logarithmic using technology.
- Focus on using key features in graphs.
- Find the characteristic of exponential and logarithmic functions.
- Solve logarithmic and exponential equations using their inverse properties and definitions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Solve problems using exponential and logarithmic functions and equations.

Model with mathematics.

- Graph exponential and logarithmic functions by hand.
- Use logarithmic functions to determine the amount of damage in an earthquake.

Use appropriate tools strategically.

- Graph exponential and logarithmic functions on a graphing calculator.

Essential questions

- What are the similarities and differences between exponential and logarithm functions?
- What is the inverse relationship between logarithmic functions and exponential functions?
- What are the similar characteristics of solving logarithmic versus exponential equations?
- How can multiple representations of logarithmic functions help you solve problems in real-world situations and/or mathematical problems?
- Why is it important to understand the opposite operation of all functions?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions	**F-IF**

Analyze functions using different representations [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions	**F-BF**

Build new functions from existing functions

F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

Students started to multiply in grade 2. Students used bar graphs and picture graphs in second grade. In grade 3, students understood the properties of multiplication and multiplied within 100. Graphing began in grade 5. Variables in equations were introduced in grade 6. In grade 8, students began their work with exponents. Functions were defined, evaluated, and compared in grade 8.

In algebra I, students used rational exponents and their properties to rewrite expressions involving radicals. Students analyzed functions (linear and quadratic) using graphs by hand and using technology, showing intercepts, maxima, and minima. Students focused on linear and exponential functions, comparing their growth rate. In algebra 1, the properties of exponents were extended to rational exponents. Exponential functions were introduced in algebra 1 and continued in algebra 2. Logarithms and inverse functions are introduced in algebra 2. In algebra 2, students graphed functions and expressed them symbolically, with and without technology.

Current Learning

Students graph equations expressed symbolically. Using the points of (1,b), (0,1) and (-1, 1\b), students graph exponential functions. Then students use a graphing calculator to graph more complex exponential equations. After demonstrating that logarithms are the inverse of exponentials, students graph logarithms with the points (b,1), (1,0), and (1\b,-1). Students will be describing the end behavior and intercepts.

Future Learning

In AP calculus AB, students will use rational exponents when calculating derivatives and antiderivatives. Exponential and logarithmic functions are used extensively throughout calculus and college algebra. Seismologists use the logarithmic functions to determine the magnitude of an earthquake. Seismic waves are checked often in the oilfields. Bankers use exponential functions for interests on savings, loans, and credit cards. Nuclear energy uses exponential decay. This was used in the development of the WIPP Site and Urenco. Archeology uses exponential functions for carbon dating of artifacts.

Additional Findings

Students sometimes struggle with the understanding of the inverse operations of exponential and logarithmic functions. “In grades 9–12, all students should understand and compare the properties of classes of functions, including exponential [and] logarithmic . . . functions.” “Students should use technological tools to represent and study the behavior of polynomial, exponential, rational, and periodic functions, among others.” (*Principles and Standards for School Mathematics*, pp. 296–297).

Precalculus, Quarter 1, Unit 1.4

Building Composite Functions

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Complete the square to change quadratic into a perfect square form.
- Derive the quadratic formula from a binomial squared.
- Use factoring and completing the square to find zeros, extreme values, and symmetry of the graph.
- Interpret the meaning of zeros, extreme values, and symmetry of the graph.
- Write compositions that describes a relationship between two quantities.

Mathematical practices to be integrated

Use appropriate tools strategically.

- Use graphing calculator to show the symmetry of the graph, the zeros, and the maximum or the minimum.

Look for and make use of structure.

- Write functions for two different quantities then compose the functions.

Essential questions

- How does the complete the square help to derive the quadratic formula?
- What is a real-life situation in which you would use to compose functions?
- Why is it important that we know how to complete the square to solve quadratic equations?

Written Curriculum

Common Core State Standards for Mathematical Content

Reasoning with Equations and Inequalities A-REI

Solve equations and inequalities in one variable

A-REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Building Functions F-BF

Build a function that models a relationship between two quantities [*Include all types of functions studied*]

F-BF.1 Write a function that describes a relationship between two quantities.★

c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Common Core Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students started to multiply in grade 2. Symmetry as an attribute of shapes was introduced in second grade. In grade 3, students understood the properties of multiplication and division. Fractions were introduced in grade 3. Operations with fractions began in grade 4. Factors are introduced in grade 4. Graphing began in grade 5. Variables in equations were introduced in grade 6.

In grade 8, students began their work with exponents. Functions were defined, evaluated, and compared in grade 8. In algebra 1, the properties of exponents were extended to rational exponents. In algebra 1, students were taught quadratic functions and factoring. In algebra 2, students will expand their repertoire of functions by working with rational and exponential expressions; polynomial, exponential, and logarithmic functions; trigonometric functions; and series. The critical areas in algebra 2 include the structural similarities between the system of polynomials and the system of integers; polynomial equations; unit circle trigonometry; and families of functions, which includes the culmination of all types of functions that have been studied with the addition of trigonometric and logarithmic functions.

Current Learning

Students will compose functions to model a relationship between to quantities. Solving quadratic functions using completing the square and by factoring is taught to help derive the quadratic formula. Factoring and completing the square process will show zeros, extreme values, and symmetry of the graph. Students interpret the zeros in terms of a context.

Future Learning

Students will use the concepts learned in calculus and college algebra. Students will need to complete the square to do translations for conics. In the area of finance, business owners use models to compare costs, revenue, and margins of profit, as well as appreciation and depreciation of goods over time. Bankers use models to compare interest rates used in calculating loans, to calculate amount of interest over time, and to compare amortization tables.

Coroners use logarithms to determine how long a body has been dead; they calculate exponential decay by determining the rate at which a body has been cooling and comparing that to temperature differences between the body and its surroundings. In both nuclear and internal medicine scientists investigate pH concentrations to determine amounts of radioactive decay as well as amounts of bacterial growth. In obstetrics, doctors determine the levels of a certain hormone that increases exponentially, and at different rates with each woman, to determine when pregnancy occurred and to predict fetus growth.

Additional Findings

"The complexity that teachers and researchers now see in function graphs flows from the fact that a graph has many potential meanings and can be interpreted in many ways . . . Both communicating and making meaning play important roles in graphing; each is appropriate in certain situations, depending on the underlying setting, the task, and the experience and knowledge of the graph user." (*A Research Companion to Principles and Standards for School Mathematics*, pp. 251–252)